

Abstract

Zero Trust Cybersecurity (ZTCS) represents a fundamental paradigm shift toward continuous verification and eliminating implicit trust. While widely adopted across industry and government, the field lacks a cohesive theoretical framework for implementing, measuring, and comparing Zero Trust Architecture (ZTA) effectiveness.

This research addresses this gap by developing a unified mathematical framework for Zero-Trust Enterprise Network Security (ZTENS) using measurement-driven theory and quantitative methods to evaluate an enterprise network ZT level.

Keywords: Zero Trust Architecture, enterprise network security, trust metrics, quantitative evaluation, security maturity models, cybersecurity education

Motivation & Problem Statement

- ZTA widely adopted in industry & government — but lacks rigorous mathematical foundations.
- No unified metric exists to measure, compare, or benchmark Zero Trust effectiveness across organizations.
- Current implementations rely on qualitative checklists rather than quantitative trust evidence.
- Cybersecurity curricula lack quantitative foundations for teaching Zero Trust principles at the graduate level.
- This work develops a formal, measurement-driven ZTENS framework to close these gaps.

Key Theoretical Contributions

- System Model Z**
Formal state-transition model $Z = (E, C, S, A, T, I, P, Y)$ defining all ZT system components.
- Session Trust Estimation**
Bayesian Beta model with decay: posterior trust $T_t = A_t / (A_t + B_t)$ updated from calibrated telemetry signals.
- Minimal-Trust System (MTS)**
Absolute zero-risk ZT is unachievable. MTS minimizes implicit trust through continuous calibrated verification.
- Zero Trust Maturity Index**
ZTMI: quantitative metric measuring proximity to minimal-trust boundary across enterprise architectures.

Education Impact

- This framework advances
 - Quantitative foundations for teaching Zero Trust at graduate level
 - Lab simulations of trust-score computation and verification policies
 - Real enterprise telemetry-based maturity assessment exercises
 - Bridges theory and practice for cybersecurity professionals

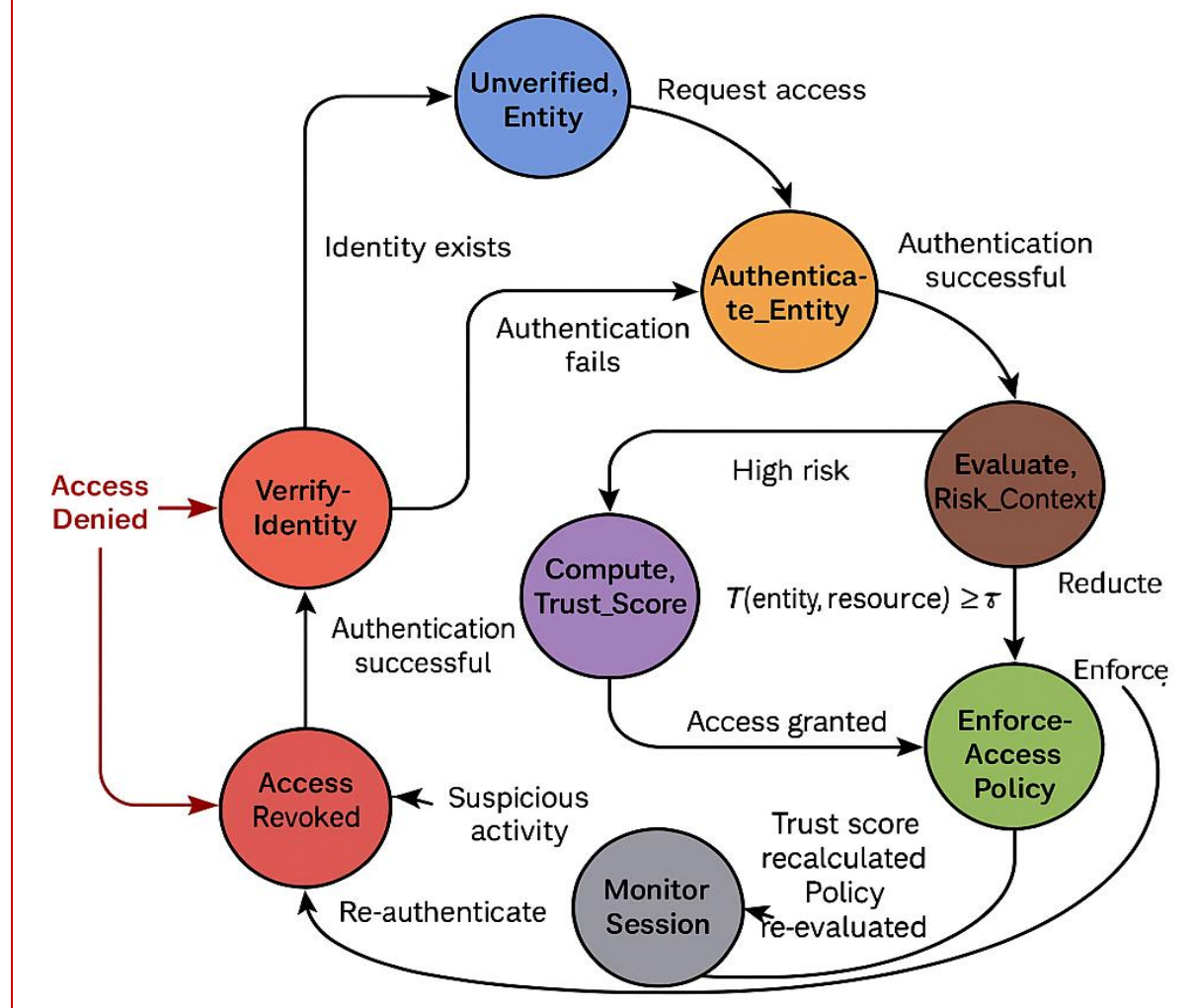
Methodology

- Formal System Modeling**
Define ZTENS as state-transition system Z with entities, context, actions, and security policy constraints.
- Trust Signal Extraction**
Telemetry $X_t \in \mathbb{R}^m$ captures identity, device, behavioral, network, environmental, and policy features.
- Calibrated Likelihood:** Logistic mapping $y_t = \sigma(b_0 + b^T X_t)$ converts raw features to probability in $[0,1]$, and maps telemetry to calibrated legitimacy likelihood.
- Bayesian Trust Update**
Beta conjugate model with exponential decay ($\rho = e^{-\lambda\delta}$) yields adaptive posterior trust T_t .
- ZTMI Scoring:** Aggregate posterior trust scores benchmarked across 10 enterprise networks.

System Model Definition — State-Transition Framework

The enterprise Zero Trust system is formally defined as a 7-tuple state-transition model: $Z = (E, C, S, A, T, I, P, Y)$

- where:
- E = Set of entities interacting within the network (e.g., Users/Roles $u \in U$, devices/edge systems $d \in D$, resources $r \in R$, sessions s).
 - C = Context/telemetry is the observed feature vector at discrete time t :
$$X_t = [X_t^{id}, X_t^{device}, X_t^{net}, X_t^{beh}, X_t^{geo}, \dots]$$
 - S = Set of possible system states (e.g., user authentication states, device management categories, access levels, network states).
 - A = Set of allowed actions (e.g., login attempts, access request types)
 - $T: S \times A \rightarrow S$ = State transition function defining how actions modify system states throughout the entire system evolution.
 - $I \subseteq S$ = Set of initial states (valid system starting configurations).
 - P = Set of security policies governing the system (e.g., continuous authentication, least privilege)
 - $\forall s \in S, \forall a \in A, T(s, a) \in P$ (This ensures all actions conform to Zero Trust principles at every stage)
 - $Y \in \{\text{legit, adversary}\}$ = Legitimacy variable



Zero-Trust Security System as a State-Transition Model

Fig. 1: State-transition model ZTENS showing identity verification, trust computation, risk evaluation, policy enforcement, and revocation in continuous ZT control.

Session-Level Latent Trust Computation & Observable Likelihood

Legitimacy is formulated as a latent variable $\theta_{s,t} \in [0,1]$ for session s at time t , and updated via Bayesian inference from telemetry evidence.

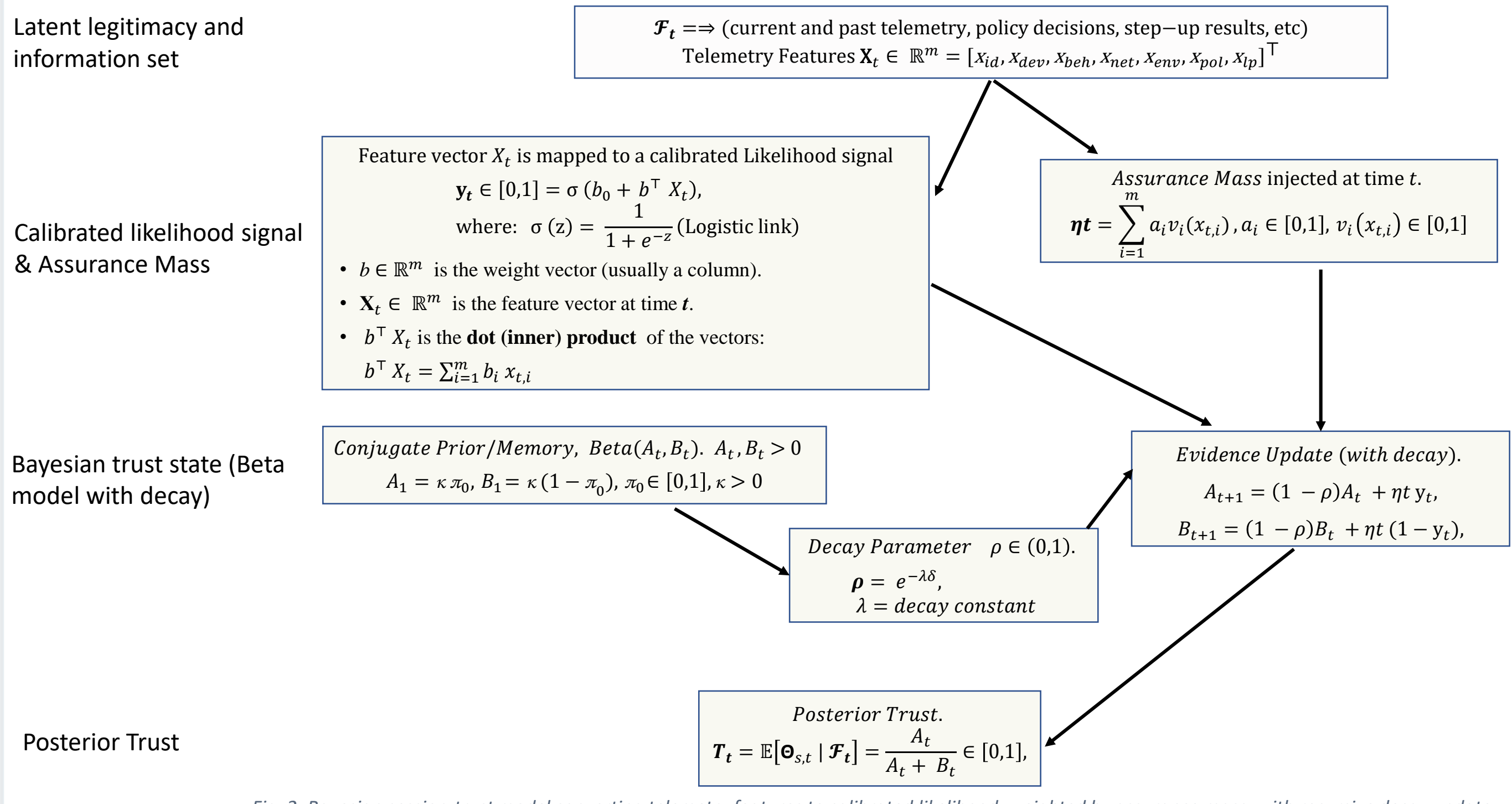


Fig. 2: Bayesian session-trust model converting telemetry features to calibrated likelihood, weighted by assurance mass, with recursive decay update.

Core Mathematical Framework

- Telemetry Feature Vector**
 $X_t = [x_{id}, x_{dev}, x_{beh}, x_{net}, x_{env}, x_{pol}, x_{ip}]^T \in \mathbb{R}^m$
- Calibrated Likelihood Signal**
 $y_t = \sigma(b_0 + b^T X_t), \sigma(z) = 1 / (1 + e^{-z})$
- Assurance Mass (Evidence Weight)**
 $\eta_t = \sum_i a_i v_i(x_{t,i}), a_i \in [0,1], v_i \in [0,1]$
- Decay Parameter**
 $\rho = e^{-\lambda\delta}, \lambda = \text{decay constant}, \delta = \text{time elapsed}$
- Bayesian Update (Beta Model)**
 $A_{t+1} = (1-\rho)A_t + \eta_t \cdot y_t$
 $B_{t+1} = (1-\rho)B_t + \eta_t \cdot (1-y_t)$
- Posterior Trust Score**
 $T_t = E[\theta_{s,t} | \mathcal{F}_t] = A_t / (A_t + B_t) \in [0,1]$

Dataset Summary

- Organizations:** 10 enterprise networks
- Sources:** IAM, EDR/XDR, SIEM, Firewall, VPN, NAC, UEBA, Policy logs
- Features:** Identity · Device · Behavior · Network · Environment · Policy
- Outputs:** Calibrated likelihood y_t · Posterior trust T_t · Enterprise ZTMI
- Observation Window:** 3–6 months per organization
- Privacy:** Pseudonymized identifiers, controlled research access

Results & Findings

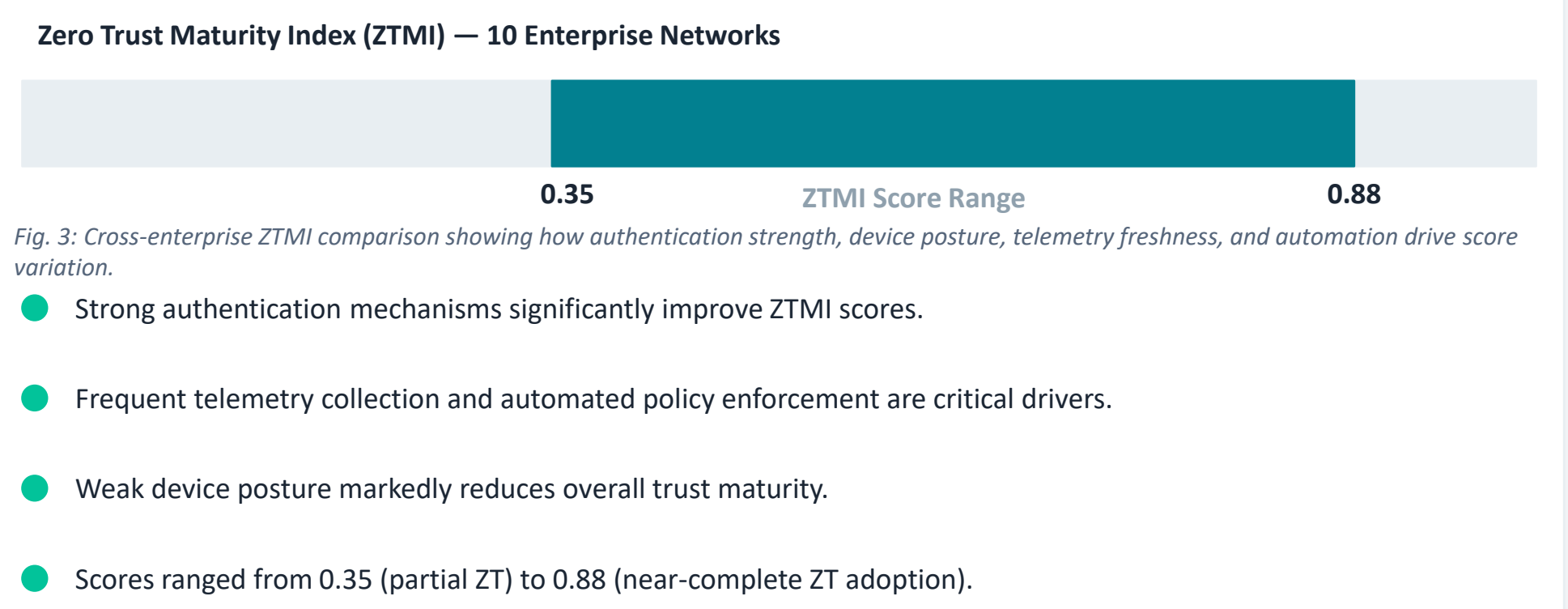


Fig. 3: Cross-enterprise ZTMI comparison showing how authentication strength, device posture, telemetry freshness, and automation drive score variation.

- Strong authentication mechanisms significantly improve ZTMI scores.
- Frequent telemetry collection and automated policy enforcement are critical drivers.
- Weak device posture markedly reduces overall trust maturity.
- Scores ranged from 0.35 (partial ZT) to 0.88 (near-complete ZT adoption).

Conclusions

This framework establishes the first unified mathematical foundation for ZTENS, enabling rigorous trust measurement, organizational benchmarking via ZTMI, and quantitative cybersecurity education. Future work will extend to multi-domain federated ZT and real-time telemetry streams.

Implementation & Data Pipeline (End-to-end research workflow from telemetry collection to trust estimation, policy decision, and enterprise maturity benchmarking.)

Goal: Transform raw enterprise telemetry into calibrated session trust scores and enterprise-level Zero Trust Maturity Index (ZTMI) values.

Telemetry Sources: Identity · Device · Network · Behavior · Environment · Policy enforcement logs

Feature Vector
 $X_t = [x_{id}, x_{dev}, x_{beh}, x_{net}, x_{env}, x_{pol}, x_{ip}]^T$ — captures authentication strength, device posture, behavioral deviation, network risk, contextual risk, and policy compliance.

Evidence Weighting: $\eta_t = \sum_i a_i v_i(x_{t,i})$ — weights each signal by source reliability and evidential strength.

Data Pipeline
Ingest → Normalize → Sessionize → Feature Engineer → Calibrate Likelihood → Bayesian Update → Policy Enforce → Aggregate ZTMI

Dynamic Trust Update
 $A_{t+1} = (1-\rho)A_t + \eta_t y_t, B_{t+1} = (1-\rho)B_t + \eta_t (1-y_t)$
 $T_t = A_t / (A_t + B_t), \rho = e^{-\lambda\delta}$ (evidence decay)

Policy Decision Logic

- 0 → 49: Low trust → Deny / Isolate
- 50 → 79: Medium trust → Step-up Auth
- 80 → 100: High trust → Allow + Monitor

$ZTMI = \sum_k w_k M_k, \sum w_k = 1$ — Enterprise Zero Trust Maturity Index

Validation Design

- Dataset Scope**
10 enterprise networks observed over a multi-month period.
- Unit of Analysis**
Each record is a session-resource decision event at time t .
- Data Preparation**
Logs parsed, deduplicated, time-aligned, sessionized, normalized into common feature schema.
- Model Evaluation**
Calibration and discrimination metrics; posterior trust evaluated on risk detection, false accepts, and adaptive policy triggering.
- Ground Truth**
Derived from analyst-reviewed incidents, policy outcomes, malware detections, anomalous activity, and threat intelligence matches.
- Comparative Benchmarking**
Framework vs. static access rules, checklist-based scoring, and uncalibrated risk models. maturity
- Privacy Controls**
User/device identifiers pseudonymized; analysis uses security metadata only.
- Expected Outcome**
Stronger auth, healthier devices, fresher telemetry → higher ZTMI and more stable trust trajectories.